Pertanika J. Sci. & Technol. 27 (2): 769 - 778 (2019)

SCIENCE & TECHNOLOGY

Journal homepage: http://www.pertanika.upm.edu.my/

Simplified APP Computation of High Order Constellations Combined with Non-Binary LDPC Decoders

Latifa Mostari[1]*, Abdennacer Bounoua[2] and Abdelmalik Taleb-Ahmed[3]

[1]*Department of Electronics, Faculty of Technology, Hassiba Benbouali University of Chlef, Algeria*
[2]*RCAM Laboratory, University of Djillali Liabès, Sidi Bel-Abbès, Algeria*
[3]*IEMN UMR CNRS 8520, Polytechnic University of Hauts-de-France, Valenciennes, France*

ABSTRACT

Combining non-binary LDPC (Low-Density Parity-Check) codes and high order constellations, such as Quadrature Amplitude Modulations (QAM), is an effective way to improve the bandwidth efficiency. Since, the message exchanged in the LDPC decoder can be measured by the APP (A Priori Probability) or the LLR (Log-Likelihood Ratio), depending on the decoding algorithm type, the message at the decoder input that computed by the QAM must be using the same calculation. However, the number of operations performed by the QAM increases with the constellation order, and the calculation changes with the channel type. In this paper, we use simplified LLR computations, introduced for binary codes, to simplify the APP calculation of square-QAM-Gray demapping for APP-based non-binary LDPC decoders. Under Gaussian channel, the simplified APP calculation achieves the same performance that obtained with the exact APP computation. The same simplified APP calculation used for a Gaussian channel can be applied, with minor operations added, for Rayleigh channel, and it shows a small performance loss with respect to the exact APP computation. These simplifications simplify the combination of non-binary LDPC codes with QAM. With this method, it is easy to change a decoding algorithm based on the APP to an algorithm based on the LLR.

ARTICLE INFO

Article history:
Received: 21 June 2018
Accepted:11 January 2019
Published: 25 April 2019

E-mail addresses:
l.mostari@univ-chlef.dz (Latifa Mostari)
nacerbounoua@yahoo.fr (Abdennacer Bounoua)
Abdelmalik.Taleb-Ahmed@univ-valenciennes.fr (Abdelmalik Taleb-Ahmed)
* Corresponding author

ISSN: 0128-7680
e-ISSN: 2231-8526

© Universiti Putra Malaysia Press

Keywords: A Priori Probability (APP), Log-Likelihood Ratio (LLR), Non-Binary LDPC code, Square-QAM-Gray mapping

INTRODUCTION

Given the increasing number of applications requiring high-speed transmission without increasing the bandwidth of the transmission channel; this is the reason for the use of high order constellations. When the constellation order is higher than eight, one uses a Quadrature Amplitude Modulation (QAM) rather than Phase Shift Keying (PSK). Therefore, the QAM is highly recommended as a high order constellation. However, communication systems using QAM require a high signal to noise ratio. To overcome this disadvantage, it is interesting to combine high error correction codes such as LDPC codes with QAM.

Binary LDPC codes that are linear block codes, can approach the Shannon limit (Shannon, 1948). They are proposed by Gallager (1962 & 1963) and rediscovered by Mackay (1999). Unfortunately, binary LDPC code shows performance degradation when the code size is small or moderate, and higher order modulations are used for transmission. Non-binary LDPC code, designed in high order Galois Fields GF(Q) where Q is the cardinality of the Galois field, is investigated by Davey and Mackay (2002) to avoid this weakness.

The LDPC decoding is done according to the principle of iterative decoding. One class of algorithms used to decode LDPC codes is the commonly known message propagation algorithms (Johnson, 2010). Message propagation algorithms are also known as iterative decoding algorithms. The first practical iterative decoding algorithm is the Sum-Product algorithm (SPA) (Gallager, 1963), also known as the belief propagation algorithm is an optimal iterative decoding algorithm but with a high computational complexity. Several algorithms have been proposed to reduce the complexity of the SPA, each one with a particular performance-complexity tradeoff.

The messages exchanged in the SPA and its versions can be measured by the APP or the LLR depending on the decoding algorithm type. Therefore, the input message of the decoder, performed by the QAM, must be computed with the same calculation that used in the decoder. However, the number of operations to calculate the LLR or the APP that introduced to the decoder, increases with the constellation order. Also, the calculation changes with the considered channel.

Several algorithms have been introduced in order to simplify the exact calculation of the LLR. The pragmatic algorithm, introduced in Le Goff et al., (1994) & Le Goff, (2000), attempts to simplify the calculation assuming that the likelihood values are Gaussian variables. The max-log-MAP (max-log Maximum A Posteriori) algorithm is the most popular simplifying the exact algorithm (Liu & Kosakowski, 2015). The simplified LLR calculations are used only for binary decoders based on LLR. This simplifications are applied for binary LDPC codes and non-binary LDPC codes (Mostari & Taleb-Ahmed, 2017 & Mostari et al., 2015) respectively. While for decoder-based on APP, we need to calculate the APP, and this latter is complex. Simplified APP computations are introduced only in Mostari et al. (2018).

The authors in Mostari et al. (2018) proposed a method for applying the simplifications of the LLR and adapted them to the binary LDPC decoder based on the APP, and even to simplify the calculation of the APP. In fact, it is easy to change a decoding algorithm based on LLR to an algorithm based on APP, while keeping unchanged the simplified calculation of LLR. The proposed method is programmed to adapt as perfectly as possible the transmission system to the channel type. Therefore, the proposed method allowing to simplify the APP calculation leads to simplify the implementation of the transmission system. In this work, we use this method for non-binary LDPC decoder based on the APP.

In this paper, we restrict our description of combining non-binary LDPC code, decoded by FFT-SPA (Fast Fourier Transform SPA) that uses APP calculations, with square Gray-QAM constellations, over Gaussian channel for satellite transmission and Rayleigh Channel for radio transmission (Barnault & Declercq, 2003; Carrasco & Johnston, 2008). Note that square QAM constellations and the others QAM constellations have different simplifications.

The rest of the paper is organized as follows: the exact APP computation, under Gaussian and Rayleigh channels, for square QAM constellations, the proposed diagram on the simplified APP calculations, the simplified APP calculation for non-binary LDPC code is given, the simulation results, discussion and concluding remarks.

EXACT APP CALCULATIONS FOR QAM WITH SQUARE CONSTELLATIONS OVER GAUSSIAN AND RAYLEIGH CHANNELS

2^m-QAM transmit, at each instant nT, m bits $\{u_{n,i}\}$, $i \in \{1, \ldots, \mathrm{m}\}$ that is represented by $\alpha_n + jb_n$, where α_n and $b_n \in \{\pm 1, \pm 3, \pm 5, \ldots, m \pm 1\}$. Binary symbols $\{u_{n,i}\}$ are obtained by the conversion of non-binary symbols to binary symbols at the output of LDPC encoder.

The simplest diagram of a digital transmission system as part of the association of an LDPC code and a 2^m-QAM, is given in Figure 1.

After passing through the transmission channel, the observation relating to the couple (a_n, b_n) is represented by a couple $(a_n^{'}, b_n^{'})$. In the case of Rayleigh channel $a_n^{'}$ and $b_n^{'}$ are given by:

$$a_n^{'} = \alpha_n a_n + z_n$$
$$b_n^{'} = \alpha_n b_n + z_n$$

where z_n is a Gaussian noise, centered, with variance σ^2 and α_n is a variable characterizes the attenuation of the transmitted signal. In the case of a Gaussian channel $\alpha_n = 1$.

At the reception, we treat the couples (a'_n, b'_n) in order to extract m samples $\{\hat{u}_{n,i}\}, i \in \{1, \dots, m\}$, that are calculated by LLR($u_{n,i}$) or APP($u_{n,i}$) .

$APP(u_{n,i})$, $i \in \{1, \dots, m\}$, is calculated as follows (Moon, 2005):

$$APP(u_{n,i} = 0) = \frac{Pr\{(a'_n, b'_n) / u_{n,i} = 0\}}{Pr\{(a'_n, b'_n) / u_{n,i} = 0\} + Pr\{(a'_n, b'_n) / u_{n,i} = 1\}}$$

$$APP(u_{n,i} = 1) = 1 - APP(u_{n,i} = 0)$$

where $Pr\{(a'_n, b'_n) / u_{n,i} = w\}$ is the probability that the available couple is (a'_n, b'_n); knowing the binary symbol $u_{n,i}$ is equal to w.

LLR($u_{n,i}$), $i \in \{1, \dots, m\}$, is calculated as follows (Alam et al., 2009):

$$LLR(u_{n,i}) = log\left[\frac{Pr\{(a'_n, b'_n) / u_{n,i} = 1\}}{Pr\{(a'_n, b'_n) / u_{n,i} = 0\}}\right]$$

This equation is the exact calculation of LLR, it is the optimal calculation that represents the log-MAP algorithm (Maximum A Posteriori). However, it involves complicated operations. Several algorithms have been introduced in order to simplify the exact calculation of the LLR such as: max-log-MAP algorithm and pragmatic algorithm.

In the case of $m=2p$, 2^{2p}-QAM modulation uses a square constellation (case of 16-QAM, 64-QAM and 256-QAM). Such modulation has the particularity to be reduced to two amplitude modulations with 2^P states independently acting on two carriers in phase and quadrature. In our work, we use a square constellation.

According to the above property of a square constellation, p expressions in the phase of the APP, for a Gaussian channel, are the following:

$$APP(u_{n,i} = 0) = \frac{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a'_n - \alpha_n a^0_{i,j}\right)^2\right\}}{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a'_n - \alpha_n a^0_{i,j}\right)^2\right\} + \sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(a'_n - \alpha_n a^1_{i,j}\right)^2\right\}}, \quad i \in \{1, \dots, p\}$$

$$APP(u_{n,i} = 1) = 1 - APP(u_{n,i} = 0), \qquad i \in \{1, \dots, p\}$$

Similarly the p relations in the quadrature path of the APP eventually lead to the following expressions:

$$APP(u_{n,i} = 0) = \frac{\sum_{j=p+1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(b'_n - \alpha_n b^0_{i,j}\right)^2\right\}}{\sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(b'_n - \alpha_n b^0_{i,j}\right)^2\right\} + \sum_{j=1}^{2^{p-1}} exp\left\{-\frac{1}{2\sigma^2}\left(b'_n - \alpha_n b^1_{i,j}\right)^2\right\}}, \quad i \in \{p+1, \dots, 2p\}$$

$$APP(u_{n,i} = 1) = 1 - APP(u_{n,i} = 0), \qquad i \in \{p+1, \dots, 2p\}$$

In the following section, we show the proposed method, that we have introduced for binary LDPC codes presented in other paper, to simplify the APP calculation.

PROPOSED BLOCK DIAGRAM OF THE SIMPLIFIED APP CALCULATIONS

In order to simplify the APP calculation, we apply the simplified LLR computation, for Gaussian and Rayleigh channels, that used for a decoder based on LLR. The simplified LLR algorithm got on a Gaussian channel can be reused efficiently on a Rayleigh channel (Figure 1), this provided to insert an additional block to accommodate, each time *nT*, the channel attenuation α_n(Le Goff, 1995).

Then, we insert an additional module to make the conversion from the LLR to the APP, as shown in Figure 2. Indeed, it is easy to change a decoding algorithm based on LLR to an algorithm based on APP, while keeping unchanged the simplified LLR calculation. Therefore, the proposed method leads to simplify the system implementation that is well shown in figure 1.

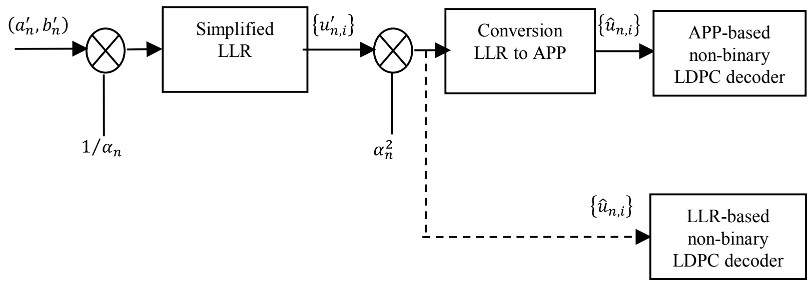

Figure 2. Principle of simplified APP calculation

Following the Figure 2, the simplification of the APP is obtained after two operations: the first is the simplification of the LLR calculation, and the second is the derivation of the APP from the LLR (Lee et al., 2005). Therefore, the derivation of the APP from the simplified LLR, leads to the simplified equations of the APP:

$$APP(u_{n,i} = 0) = \frac{1}{1+exp\left(LLR\left(u_{n,i}\right)\right)}, \qquad i \in \{1, \dots, 2p\}$$

$$APP(u_{n,i} = 1) = 1 - APP(u_{n,i} = 0), \qquad i \in \{1, \dots, 2p\}$$

where LLR($u_{n,i}$) represents the simplified calculation of the LLR. In our work, we use max-log-MAP algorithm and pragmatic algorithm :

Max-log-MAP Algorithm

The Max-log-MAP algorithm, introduced in Liu and Kosakowski (2015) shows that the *p* relations in the phase and *p* relations in the quadrature are given respectively by the following equations:

$$LLR(u_{n,i}) = \frac{\left(\min_{j\in\{1,\dots,2^{p-1}\}}\left(a'_n - a^0_{i,j}\right)\right)^2 - \left(\min_{j\in\{1,\dots,2^{p-1}\}}\left(a'_n - a^1_{i,j}\right)\right)^2}{2\sigma^2}, \; i \in \{1, \dots, p\}$$

$$LLR(u_{n,i}) = \frac{\left(\min_{j\in\{1,\dots,2^{p-1}\}}\left(b'_n - b^0_{i,j}\right)\right)^2 - \left(\min_{j\in\{1,\dots,2^{p-1}\}}\left(b - b^1_{i,j}\right)\right)^2}{2\sigma^2}, \; i \in \{p+1, \dots, 2\,p\}$$

Pragmatic Algorithm

The pragmatic algorithm introduced in LeGoff et al. (1994) shows that the *p* relations in the phase and *p* relations in the quadrature, are given respectively by the following equations:

$$LLR(u_{n,1}) = a'_n$$
$$LLR(u_{n,2}) = -|LLR(u_{n,1})| + 2^{p-1}$$
$$\vdots$$
$$LLR(u_{n,i}) = -|LLR(u_{n,i-1})| + 2^{p-i+1}$$
$$\vdots$$
$$LLR(u_{n,p}) = -|LLR(u_{n,p-1})| + 2$$

And

$$LLR(u_{n,p+1}) = b'_n$$
$$LLR(u_{n,p+2}) = -|LLR(u_{n,p+1})| + 2^{p-1}$$
$$\vdots$$
$$LLR(u_{n,p+i}) = -|LLR(u_{n,p+i-1})| + 2^{p-i+1}$$
$$\vdots$$
$$LLR(u_{n,2p}) = -|LLR(u_{n,2p-1})| + 2$$

Therefore, it is remarkable that the simplified APP calculation are less number of operations that the exact APP calculation.

SIMPLIFIED APP CALCULATION FOR NON-BINARY LDPC CODES

For non-binary LDPC decoding algorithms based on APP, defined in a Galois Field $GF(2^q)$, the exchanged message is the APP calculated on non-binary symbols *a*, $a \in GF(2^q)$, $APP(a)$, that is calculated as follows:

$$APP(a = v) = Pr\{a = v/(a'_n, b'_n)\}$$

Where $Pr\{a = v/(a'_n, b'_n)\}$ represents the probability that the symbol *a* transmitted has a value *v*, $v \in GF(2^q)$, knowing that the available couple at the channel output is (a'_n, b'_n).

Each non-binary symbol *a*, $a \in \{0, 1, \dots, 2^q - 1\}$, corresponds to the binary sequence $\{u_{n,1}, u_{n,2}, \dots, u_{n,q}\}$ in *GF*(2). Therefore, the equation (22) becomes:

$$APP(a = v) = \prod_{i=1}^{q} Pr\{u_{n,i} = w/(a'_n, b'_n)\}$$

where the binary value *w* is associated to the value *v*.

Therefore, using Bayes rule, the precedent expression becomes:

$$APP(a = v) = \prod_{i=1}^{q} APP(u_{n,i} = w)$$

SIMULATION RESULTS

In this section, we will show the effect of the simplified calculation of the APP on the performance of non-binary LDPC codes constructed on GF(4) of rate equals to 1/2 and a parity check matrix of size 512 × 1024 (4-ary (512, 1024) LDPC Code).

A decoding algorithm using the FFT-SPA where the number of iterations is four. The transmission chain for which we evaluated the Binary Errors Rate (BER) after the decoding used a non-binary LDPC code, attached to two square constellations: 16-QAM and 64-QAM, using Gray mapping, and two transmission channels: Gaussian and Rayleigh.

Results, obtained by computer simulations, are given in terms of Bit Error Rate (BER) versus E_b/N_0, where E_b is the energy per information and N_0 is the spectral density noise.

Figure 3 shows performance comparisons, on a Gaussian channel, between a non-binary LDPC code using the exact calculation of the APP, a non-binary LDPC code using the simplified calculation by applying the pragmatic algorithm and a non-binary LDPC code using the simplified calculation by applying the max-log-MAP algorithm.

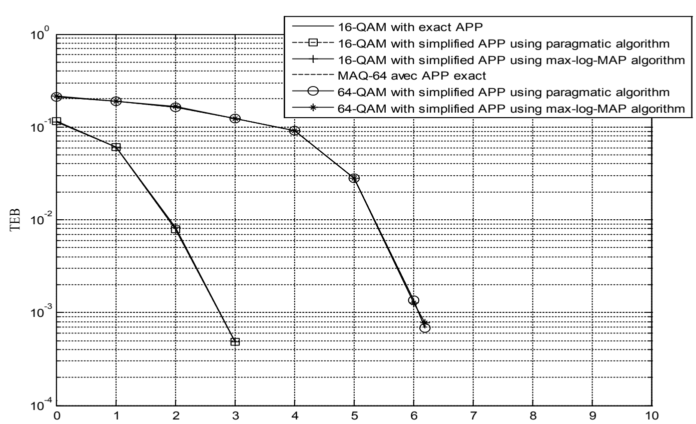

Figure 3. Performance comparisons, under a Gaussian channel, of 4-ary (512, 1024) LDPC code decoded by FFT-SPA using exact APP and simplified APP algorithms of 16-QAM and 64-QAM

In order to study the influence of the simplified calculation on the performance of a non-binary LDPC code on a Rayleigh channel, the same performance comparison of figure 3 are performed on a Rayleigh channel, in Figures 4 and 5.

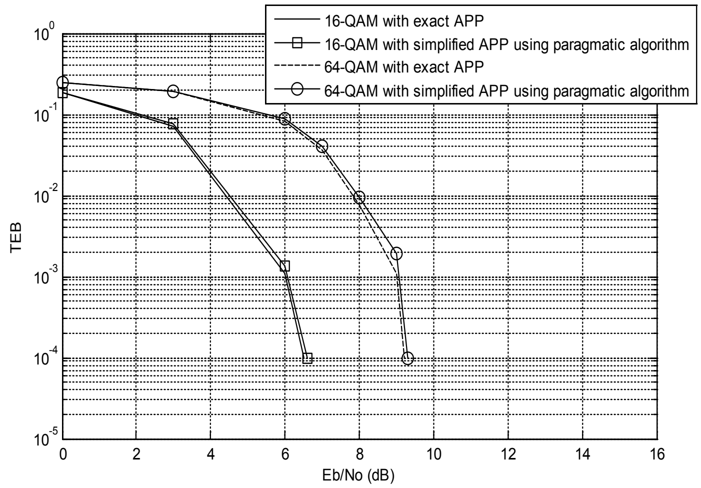

Figure 4. Performance comparisons, under a Rayleigh channel, of 4-ary (512, 1024) LDPC code decoded by FFT-SPA using exact APP and simplified APP algorithms, using pragmatic algorithm, of 16-QAM and 64-QAM

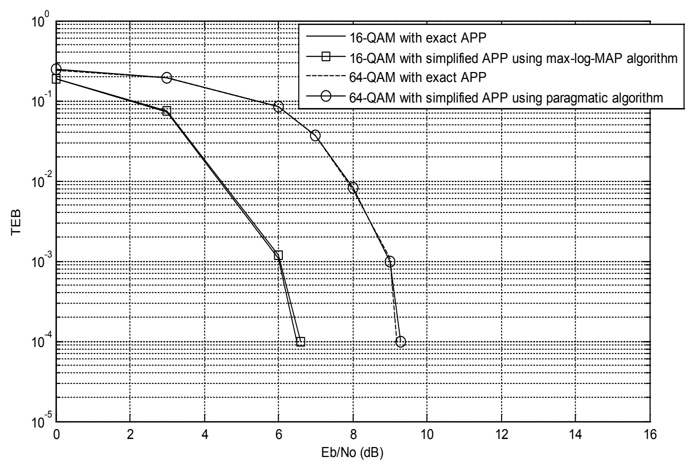

Figure 5. Performance comparisons, under a Rayleigh channel, of 4-ary (512, 1024) LDPC code decoded by FFT-SPA using exact APP and simplified APP algorithms, using max-log-MAP algorithm, of 16-QAM and 64-QAM

DISCUSSION

Figure 3 shows that the simplified calculation of the APP, on a Gaussian channel, using a max-log-MAP algorithm and a pragmatic algorithm, has no effect on the performance of a non-binary LDPC code.

Under Rayleigh channel, as seen in Figures 4 and 5 respectively, in comparison with the exact APP computation of 16-QAM and 64-QAM associated with non-binary LDPC codes, the simplification of the APP computation, with pragmatic algorithm, has a small performance loss, and the simplification of the APP computation, with max-log-MAP algorithm, has a very small performance loss at high E_b/N_0.

The results of our simulation are generalized with small or high order square-QAM constellations. As a result, the simplification of the APP calculation can achieve a good performance with a simple computation. It leads to simplify the system implementation.

CONCLUSION

In this work, we used the simplified LLR calculation, introduced for binary code, to facilitate the APP calculation for non-binary LDPC codes. The proposed method for making these simplifications, puts a system combining a non-binary LDPC code and a high order constellation simple to implement. It is programmed to adapt as perfectly as possible the system to the type of channel in question and to the type of decoding algorithm. Also, it ensures an efficient decoding regardless of the channel type. Therefore, since LDPC codes are selected as candidate for 5th generation wireless communications (5G), it is essential to develop a new technique allowing the simplification of the transmission system for 5G and Satellite communication systems.

REFERENCES

Alam, M. Z., Islam, C. S., & Sobhan, M. A. (2009). Using Log Likelihood Relation for BER Analysis of QAM in Space Diversity. *Journal of Communications*, 4(6), 371-379.

Barnault, L., & Declercq, D. (2003, March 31 - April 4). Fast Decoding Algorithm for LDPC over GF(2*q)*. In *Proceedings 2003 IEEE Information Theory Workshop (Cat. No. 03EX674)* (pp. 70-73). Paris, France.

Carrasco, R. A., & Johnston, M. (2008). *Non-binary error control coding for wireless communication and data storage*. Chichester: John Wiley & Sons.

Davey, M., & MacKay, D. (2002). Low-Density Parity Check Codes over GF(*q*). *IEEE Communications Letters, 2*(6), 165-167.

Gallager, R. G. (1962). Low-Density Parity-Check Codes. *IEEE Transaction Information Theory, 8*(1), 21-28.

Gallager, R. G. (1963). *Low-density parity-check codes*. Cambridge: MIT Press.

Johnson, S. J. (2010). *Iterative error correction: Turbo, low-density parity-check and repeat-accumulate codes*. Cambridge: Cambridge University Press.

Le Goff, S. Y. (2000). Bit-interleaved turbo-coded modulations for mobile communications. In *2000 10th European Signal Processing Conference* (pp. 1-4). Tampere, Finland.

Le Goff, S. (1995). *Les turbo-codes et leur application aux transmissions à forte efficacité spectrale* [Turbo-Codes and their Applications to Transmission with High Spectral Efficiency] (Doctoral dissertation). University of Western Brittany, Brest.

Le Goff, S., Glavieux, A., & Berrou, C. (1994). Turbo-codes and high spectral efficiency modulation. In *Proceedings of ICC/SUPERCOMM'94-1994 International Conference on Communications* (pp. 645-649). New Orleans, LA, USA.

Lee, S. H., Seok, J. A., & Joo, E. K. (2005). Serially concatenated LDPC and turbo code with global iteration. In *18th International Conference on Systems Engineering (ICSEng'05)* (pp. 201-204). Las Vegas, NV, USA.

Liu, X., & Kosakowski, M. (2015). Max-Log-MAP Soft Demapper with Logarithmic Complexity for M-PAM Signals. *IEEE Signal Processing Letters, 22*(01), 50-53.

MacKay, D. (1999). Good Error-Correcting Codes Based on very Sparse Matrices. *IEEE Transaction Information Theory, 45*(2), 399-431.

Mostari, L., & Taleb-Ahmed, A. (2017). Simplified Log Likelihood Ratio for Binary LDPC Codes. *Journal of Engineering Science and Technology Review, 10*(1), 37-40.

Mostari, L., Taleb-Ahmed, A., & Bounoua, A. (2015). Simplified soft Output Demapper for Non-Binary LDPC Codes. *Optik- International Journal for Light and Electron Optics*, *126*(24), 5075-5076.

Mostari, L., Taleb-Ahmed, A., & Bounoua, A. (2018). Simplified a posteriori probability calculation for binary LDPC codes. *Journal of Science and Technology, 26*(4), 1751-1763.

Moon T. K. (2005). *Error Correction Coding: Mathematical Methods and Algorithms*. New Jersey: A John Wiley & Sons, Inc.

Shannon, C. E. (1948). A Mathematical Theory of Communication. *Bell System Technical Journal, 27*(3), 379-423.